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Maximum Likelihood Trajectory Determination from Radar Data

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MAXIMUM LIKELIHOOD TRAJECTORY DETERMINATION FROM RADAR DATA

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Group 35

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ABSTRACT

Analysis is given for accurately determining a target's trajectory from radar data using maximum likelihood techniques. The method is incorporated in a computational program and obtains converged trajectories in 1 to 3 iterations when applied to satellite track data from the TRADEX radar. Several tracks with differing signal-to-noise ratios are presented. The average RMS residuals obtained for 300 to 450 seconds of data are 25 feet in range and 0.05 degrees in elevation and azimuth, while some tracks at high S/N resulted in angle residuals of about .01 degrees, equal to the pointing accuracy of TRADEX.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

MAXIMUM LIKELIHOOD TRAJECTORY DETERMINATION FROM RADAR DATA

I. INTRODUCTION

A problem of considerable importance in the study of radar tracking data is how best to obtain a target's trajectory taking into account the relative accuracy of range and/or doppler measurements over the coarser angle measurements. Various formulations of this problem have been made and this report draws upon these previous works. This report describes a maximum likelihood determination of the unknown parameters defining a target's trajectory. Illustrations of the accuracy and speed of the method are made with examples of fitted satellite data from the TRADEX radar. The procedure obtains a solution in 3 iterations or less.

Basically, the analysis contains these important elements. The method is a general maximum likelihood method in that it obtains a reference trajectory from the data and then perturbs the parameters, in this case, the six initial conditions, to produce a time history of the resulting changes in the trajectory. These differences can be viewed as a manifold of linearly independent functions of time. The data residuals are assumed to be members of this manifold and the components which give the proper corrections to the initial conditions are uniquely determined by maximization of a likelihood expression.

Section II gives a description of the techniques with a basically complete but simplified derivation of the maximum likelihood algorithms. Section III contains illustrations of the accuracy and fast convergence of the method with curves and tables of results from TRADEX radar data. An interesting study of how the trajectory initial conditions deteriorate as shorter data spans are used is given. Section IV contains some illustrative cases where, starting with elevation and range data without refraction corrections, the method obtains residuals which roughly agree with the normal values for refraction correction. Section V presents the conclusions from this study and suggests other areas for further investigations. An appendix gives a derivation of a refraction correction formula used in the data reduction.

II. DESCRIPTION OF TECHNIQUE

The technique will first be described in an abstract way and then the procedure used by the computational program will be outlined in simpler terms. In general, the equations of motion of the target are supposed to be known, except for a certain number of constants or parameters (P_k). In the case of the satellite problem, with the gravitational model uniquely defined, the only parameters are the six initial conditions; and, henceforth, we set $k = 1,6$. The analysis then proceeds along the following lines:

A reference trajectory is obtained from a set of zeroth-order parameters (P_k^o). Any neighboring trajectory has its parameters incremented by a 6-component vector (x_k), which is to be added to (P_k^o). The differences between a neighboring trajectory and the reference one, in the radar observables range r, elevation e, and azimuth a, are functions of time t. Let $(x_k)_n$, n = 1,6, be six different increment vectors defined successively by the component arrays: (1,0,0,0,0,0), (0,1,0,0,0,0) ....(0,0,0,0,0,1). One then computes a set of eighteen (6 by 3) "influence functions", which represent the perturbations $\delta r_n(t)$, $\delta e_n(t)$, and $\delta a_n(t)$, n = 1,6, due to the unit vector increments in one of the six parameters. The perturbations corresponding to parameter increments given by the array $(X_1, X_2, X_3, X_4, X_5, X_6)$ are then given by $\delta r(t) = \sum_{n=1}^{6} X_n \delta r_n(t)$, in which the linearity in the coefficients (X_n) is a basic assumption for the maximum likelihood method. Any departure from linearity can be rectified by iteration. Similar expressions are defined for $\delta e(t)$ and $\delta a(t)$. The time series of radar data minus the reference trajectory gives the three data residuals $\delta R(t)$, $\delta E(t)$, and $\delta A(t)$. The deviation $\Delta r(t)$, defined as $\delta r(t) - \delta R(t)$, is still linear in (X_n); similarly defined are $\Delta e(t)$ and $\Delta a(t)$. Under the assumption of a normal distribution in the errors, a likelihood expression $L(X_n)$ is then defined as the sum of the squares of these residuals divided by their respective standard deviations. Referring to N data points and to standard deviations σ_r, σ_e, and σ_a for the

radar observables we have:

$$L(X_n) = - \sum_{t=t_o}^{t_N} \{[\Delta r(t)/\sigma_r]^2 + [\Delta e(t)/\sigma_e]^2 + [\Delta a(t)/\sigma_a]^2\}^*$$

Maximization of $L(X_n)$ yields a set of 6 normal equations to determine (X_n).

The procedure used in the computational program based on the analysis is as follows:

1. A data set of N points is given along the trajectory. A smaller set m of these points is fitted in r, e, and a with polynomials to obtain starting initial conditions.

2. The equations of motion are integrated with these initial conditions to obtain a reference trajectory.

3. The six initial conditions are then perturbed one at a time and new trajectories are obtained. Differences in r(t), e(t), and a(t) are formed by subtracting the reference trajectory values from those of the perturbed trajectories. These differences are fitted with polynomials to form a polynomial coefficient matrix P(i,j,n). The subscript i from 1 to 3 refers to range, elevation, or azimuth, j refers to the degree of the polynomial up

*The inclusion of doppler data deviations is straightforward but is not retained here for simplicity.

to a maximum of J, and n to the species of the perturbed initial condition. Fourth order polynomials are used for range residuals and second order polynomials for the angles.

4. Data residuals are obtained by subtracting the reference trajectory from the data. These data residuals are fitted with polynomials to form the coefficient matrix Q(i,j). The likelihood expression $L(X_n)$ is then formed as follows:

$$L(X_n) = \sum_{i=1}^{3} \frac{1}{\sigma_i^2} \left[\sum_{n=1}^{6} X_n \left(\sum_{j=0}^{J} P(i,j,n) t^j \right) - \sum_{j=0}^{J} Q(i,j) t^j \right]^2$$

5. The partial derivatives of $L(X_n)$ with respect to the 6 X's are set to zero to give the system of six linear equations for determination of the X's which in turn give the corrected initial conditions and an improved trajectory. If the resulting data residuals are still large the improved trajectory becomes the reference for the next iteration.

6. As the iterations continue only the data residual polynomials used to form the new Q(i,j) are computed while the recomputation of P(i,j,n) is usually not necessary. The iterations cease when the corrections converge to negligible values. The RMS residuals of data minus the improved trajectory also decrease with every iteration. These RMS residuals, rather than the small corrections, give a figure of merit of the present method.

In the different stages of analysis and numerical computation, different coordinate systems are adopted either for logical reasons or for convenience. The particular coordinate system used in the analysis of the likelihood expression and initial condition increments consists of the usual range, elevation, and azimuth for position. However, for the velocity part of the 6-vector giving the corrections to the initial conditions, it has been found useful and instructive to adopt a rectangular coordinate system of special orientation. The three axes point in the direction of (a) the range rate velocity, (b) the remaining component of the velocity perpendicular to the range rate, and (c) the third component orthogonal to (a) and (b). In other words the P_6^o component of the parameter vector is defined to be zero. This choice of velocity directions is useful in that the range rate is usually known quite accurately from range data even if doppler data is not available. Thus, the velocity part of the analysis is basically a determination of the errors in the two components perpendicular to the range rate.

III. APPLICATION TO DATA

To illustrate the foregoing procedure, some examples of fitted satellite data from the TRADEX radar will be given. The computational program developed uses a high-order Runge-Kutta integration and a gravitational model incorporating spherical harmonic terms.

The range and elevation radar data from a typical satellite track of 450 seconds are given in Figures 1 and 2. The data were corrected for refraction before analysis. After three iterations the trajectory residuals in range, doppler*, elevation and azimuth are shown in Figures 3 to 6. These figures illustrate the quantitative nature with which the derived trajectory follows the data up to quantities all of the order of the noise variations of the radar observables. This is particularly remarkable in the case of the range where the departure of the trajectory from the data remains within the order of 25 feet for a long time span of 400-500 seconds, during which time the target has traveled a distance of over 10 million feet and the range varied by several million feet. This case is typical of the many cases analyzed, 10 of which are summarized in Table 1, each exhibiting small RMS residuals.

A study was made to determine how constant the trajectory initial conditions remain as shorter data spans are used. An example was chosen which appeared to be an average case. First, 290 seconds of data (1 point per second) were used and the initial conditions obtained for this trajectory are given in Table 2. The velocity components given are a standard rectangular set. Table 3 contains the velocity differences for 3 other cases using

*The UHF analog doppler data is just shown for illustration; it was not used in the likelihood expression. The sporadic noise component was due to equipment problems and has been eliminated.

shorter data spans. These velocity differences, which can be interpreted as errors in the initial velocity estimates, increase monotonically as the data span is shortened, becoming worst for the 50-second case. On the other hand, Table 4 indicates that the values for the RMS residuals remain nearly constant for all the cases. This apparent contradiction is explained by the fact that to obtain the same RMS values the shorter span of data does not require as much accuracy in the determination of the initial velocity; while the smaller velocity differences obtained for the longer spans give a measure of the accuracy needed in the velocity determination in those cases. Note that the initial range rate receives only minor changes. Thus, the main changes are in the components orthogonal to the range rate.

We shall use the iteration history of the changes in the initial velocity for two cases to illustrate some remarks about the speed of convergence and the special velocity coordinate system mentioned previously. Table 5 contains the corrections in the initial conditions from one iteration to the next for the first case. The procedure basically converged in one trial, and this fact indicates that the assumption of linearity of the perturbations relative to the corrections is a valid one. The case in Table 5 is one for which a large component of the initial velocity is in the range rate direction, and this tends to simplify the determination of the initial velocity. In Table 6, a

case is shown for which the initial range rate is a very small component of the initial velocity. In this case, the procedure needed three iterations for convergence but again the largest corrections were in the velocity components orthogonal to the range rate velocity. For further illustration, the second and third iteration trajectories of this last case will be given to show how sensitive the range fit is to velocity errors. Figure 7 contains the range residuals after the second iteration and Figure 8, the residuals after the third iteration. The additional velocity correction introduced after the second iteration is only about 1.5 feet per second; yet this change accounts for the difference in the two fits. The RMS residual values for the two cases are given in Table 7. The determination of the initial velocity to an accuracy of one foot per second implies a very high accuracy in determining the initial angle rates. Since angle rate is not a measured quantity, ordinary angle smoothing methods could not derive these rates with an accuracy comparable to the global fitting technique used here.

IV. REFRACTION CORRECTION ANALYSIS

Consideration was given to ascertaining the importance of applying refraction corrections to elevation data, especially at low angles. An attempt was made to determine if the refraction corrections could be obtained as residuals from an analysis of uncorrected data. The results of this effort were quite

satisfactory. Shown in Figure 9 are the elevation residuals obtained from uncorrected data in a case which contained low elevation angles. In the same figure, tabulated refraction corrections are plotted, and these values essentially agree with the residual curves. Additional results are shown in Figure 10 from another satellite track. For the cases analyzed which began at low elevation, it was found necessary to start the integration from a high elevation to obtain satisfactory convergence. This fact coupled with the residuals obtained when no refraction corrections are applied point out the necessity for applying refraction corrections to radar data to obtain accurate trajectories.

A set of closed formulas for refraction correction was derived for the elevation data. The essentials of the derivation are contained in the appendix. The numerical results agree with the tabulated values.

V. CONCLUSIONS AND DISCUSSION

1. A technique to obtain accurate maximum likelihood trajectories has been developed and applied to radar satellite data with excellent results.

2. The measure of the goodness of the trajectory's fit to the data lies on one hand in the final RMS values of the residuals in range and angles and on the other hand in the requirement that the fit prevails over as long a time span as practicable. The

typical horizon-to-horizon track is of the order of 500 seconds, and for these spans RMS values of 25 feet in range and 0.05 degrees in angle have been obtained.

3. The best initial conditions, especially in velocity, are obtained after a number of iterations to determine the corrections to these conditions. In practice, one to three iterations were found to be necessary for convergence, and this depended on whether the target was initially approaching head-on (faster convergence) or in a broadside orientation (slower).

4. The repeated iterations essentially obtain corrections to the two velocity components perpendicular to the range rate velocity. The magnitudes of these components can be of the order of hundreds of feet/second in the first iteration down to one foot/second on the final iteration.

5. It is essential to apply the refraction corrections to elevation angle data in cases of low elevation angles. In addition, if the target appears first at low elevation, it is found necessary to start the integration from the high elevation angle end to obtain reasonable convergence.

This study and the availability of the related computational program presents a number of possible extensions. One area of study would be to extend the trajectory to the next pass of the satellite and compare it with actual radar observations. Such a comparison would be extremely informative and is currently being developed.

The permissible approximation of the 18 influence functions of time by a finite polynomial coefficient matrix P(i,j,n) with less than 200 elements suggests that further study should be made to determine if these P(i,j,n) elements can in turn be approximated by simple functions of the initial conditions. This in turn would suggest the exploration of applying the method to real-time trajectory determination.

Another extension could be to study the determination of a ballistic coefficient curve which is described by a few parameters just as this method has been applied to finding the 6 initial conditions describing a target's trajectory.

APPENDIX

APPROXIMATE FORMULAS FOR REFRACTION CORRECTIONS IN AN EXPONENTIAL ATMOSPHERE

Refraction corrections in elevation under a single exponential refractivity model are usually obtained by careful numerical integration and tabulated as a function of range and elevation. It is fairly cumbersome to incorporate this table into a computer program. We therefore have derived a set of approximate formulas for the elevation correction as a function of range, elevation and the two parameters α and B defining the exponential refractivity model.

1. Notation

R = Range, in units of earth's radius

ϵ = Apparent elevation

Ψ = True elevation

H = Altitude, in units of earth's radius, as a function of R and Ψ

n = Radio refractive index = $1 + \alpha \exp(-BH)$

$\Delta\epsilon = \epsilon - \Psi$ = Refraction correction for elevation

2. Formula I: For elevation < 4.5°

$$\Delta\epsilon(\text{radians}) = \alpha\,(\operatorname{Cos}\Psi(1 + \operatorname{Sin}\Psi/R)(\pi B/2)^{\frac{1}{2}} \exp(B \operatorname{Sin}^2 \psi/2) \left\{\operatorname{erf}[(B/2)^{\frac{1}{2}}(R + \operatorname{Sin}\psi)] - \operatorname{erf}[(B/2)^{\frac{1}{2}} \operatorname{Sin}\psi]\right\}$$

3. Formula II: For elevation > 4.5°

$$\Delta\epsilon(\text{radians}) = \alpha(\text{Cos}\ \psi + \text{Cos}\ \psi\ \text{Sin}\ \psi/R)$$
$$\Big\{[1-(1/B)(1 + 1/B)/\text{Sin}^2\ \psi]/\text{Sin}\ \psi$$
$$- \exp\ (-BH)[1-(1/B)(1 + 1/B + H)/(R + \text{Sin}\ \psi)^2]/$$
$$(R + \text{Sin}\ \psi)\Big\} + \alpha[\exp\ (-BH)\ -1]\ \text{Cos}\ \psi/R$$

4. To obtain a smooth function of the elevation that will bridge over the transition at 4.5°, one could use a linear interpolation formula between I and II for the interval of say, $4^{\circ} < \epsilon < 5^{\circ}$.

5. The above set of formulas has an error of less than one percent as compared to accurate numerical integration results. Since the formulas have been derived in terms of the unknown true elevation ψ, an iterative procedure has to be used, starting with ϵ in the place of ψ in the formulas. Two iterations are found to be sufficient.

TABLE I

SUMMARY OF FITTED TRAJECTORIES

Test	Time Span seconds	Range RMS feet	Elevation RMS degrees	Azimuth RMS degrees
33	450	20.9	0.0404	0.0575
34	290	16.0	0.0423	0.0766
35	420	26.9	0.0132	0.0127
36	360	19.8	0.0114	0.0107
37	320	20.3	0.0220	0.0501
38	450	51.2	0.0604	0.0594
39	450	23.1	0.0114	0.0094
40	450	18.0	0.0394	0.0537
600A	450	20.2	0.0752	0.0733
600B	450	34.1	0.0836	0.0828

TABLE II

INITIAL CONDITIONS FOR TEST 34 USING 290 SECONDS

Range feet	Doppler ft/sec	Azimuth degree	Elevation degree	V_x ft/sec	V_y ft/sec	V_z ft/sec
5140401.	-8366.1	269.9454	42.5830	13103.83	-18495.50	1875.75

TABLE III

DIFFERENCES IN INITIAL CONDITIONS OBTAINED BY USING SHORTER DATA SPANS

$$\Delta IC = IC_N - IC_{290}$$

N	ΔR	$\Delta \dot{R}$	ΔAZ	ΔE	ΔV_x	ΔV_y	ΔV_z
seconds	ft	ft/sec	degree	degree	ft/sec	ft/sec	ft/sec
200	3	-0.1	-0.0034	-0.0012	-1.02	-0.85	0.27
100	-28	1.4	-0.0089	-0.0037	-2.95	1.94	3.03
50	-3	0.0	+0.0062	-0.0044	17.42	13.74	18.00

TABLE IV

RMS VALUES FOR CASES CONTAINED IN TABLES II AND III

N seconds	Range feet	Azimuth degree	Elevation degree
290	16.0	0.0766	0.0423
200	15.9	0.0712	0.0375
100	23.3	0.0783	0.0434
50	16.2	0.0756	0.0465

TABLE V

ITERATION HISTORY OF CHANGES IN INITIAL VELOCITY FOR LARGE RANGE RATE CASE

Iteration	$\Delta V_{\dot{R}}$ ft/sec	$\Delta V \perp$ to $\dot{R}$ ft/sec	$\Delta V \perp$ to both ft/sec
0	-4.11	-25.84	-29.42
1	0.13	0.15	0.18
2	0.00	0.00	0.01
3	0.00	0.00	0.00
Total Velocity	13657.	17798.	0.00

TABLE VI

ITERATION HISTORY OF CHANGES IN INITIAL VELOCITY FOR SMALL RANGE RATE CASE

Iteration	$\Delta V_{\dot{R}}$ ft/sec	$\Delta V_{\perp}$ to $\dot{R}$ ft/sec	$\Delta V_{\perp}$ to both ft/sec
0	29.75	992.91	153.75
1	-12.11	-3.01	29.27
2	-0.38	-0.40	1.29
3	0.00	-0.02	0.02
Total Velocity	1530.00	23726.00	0.00

TABLE VII

RMS VALUES FOR 2 ITERATIONS FOR SMALL RANGE RATE CASE

Iteration	Range	Azimuth	Elevation
	feet	degree	degree
2	37.61	0.0756	0.0738
3	20.37	0.0757	0.0737

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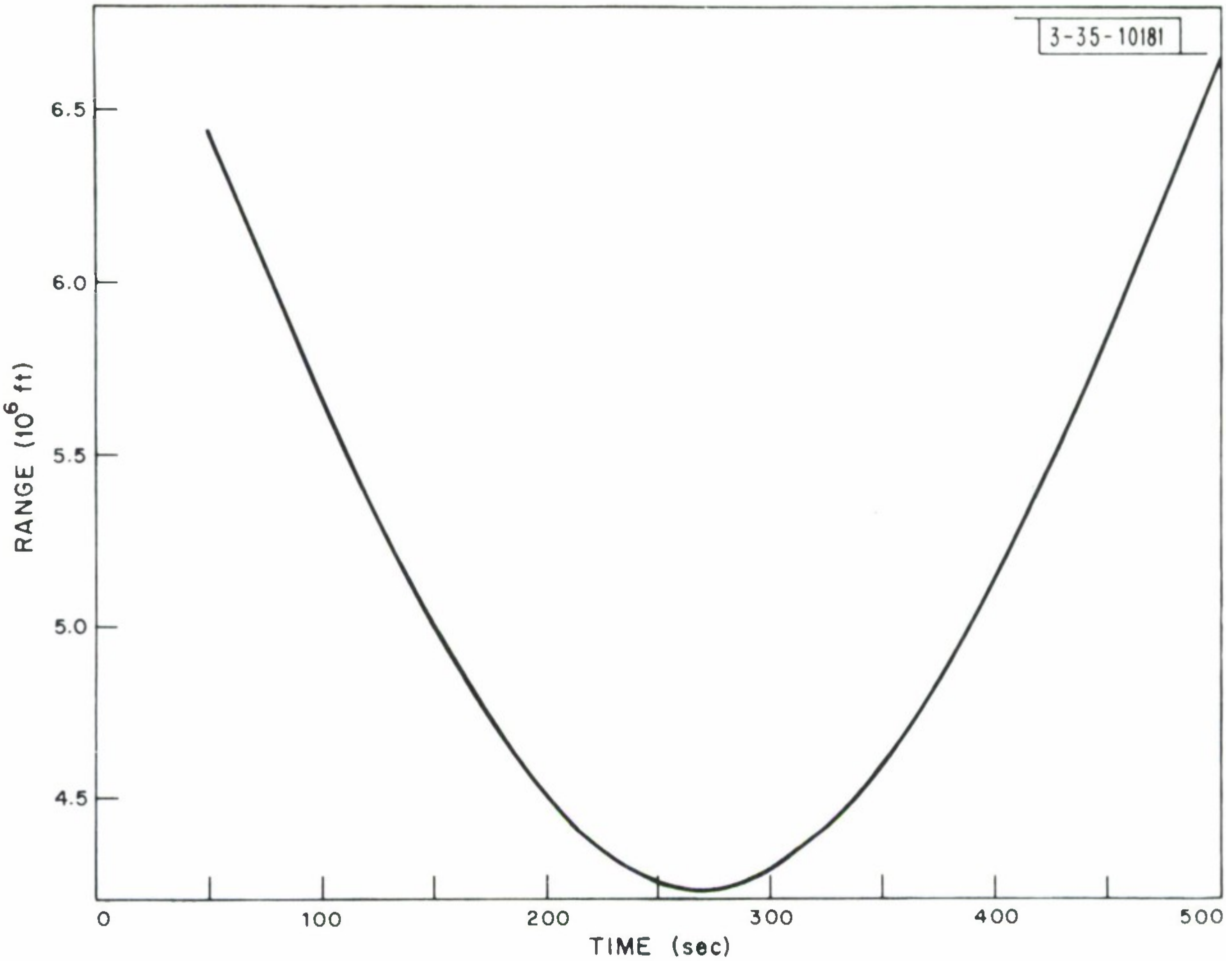

Figure 1: Range versus time for test 33.

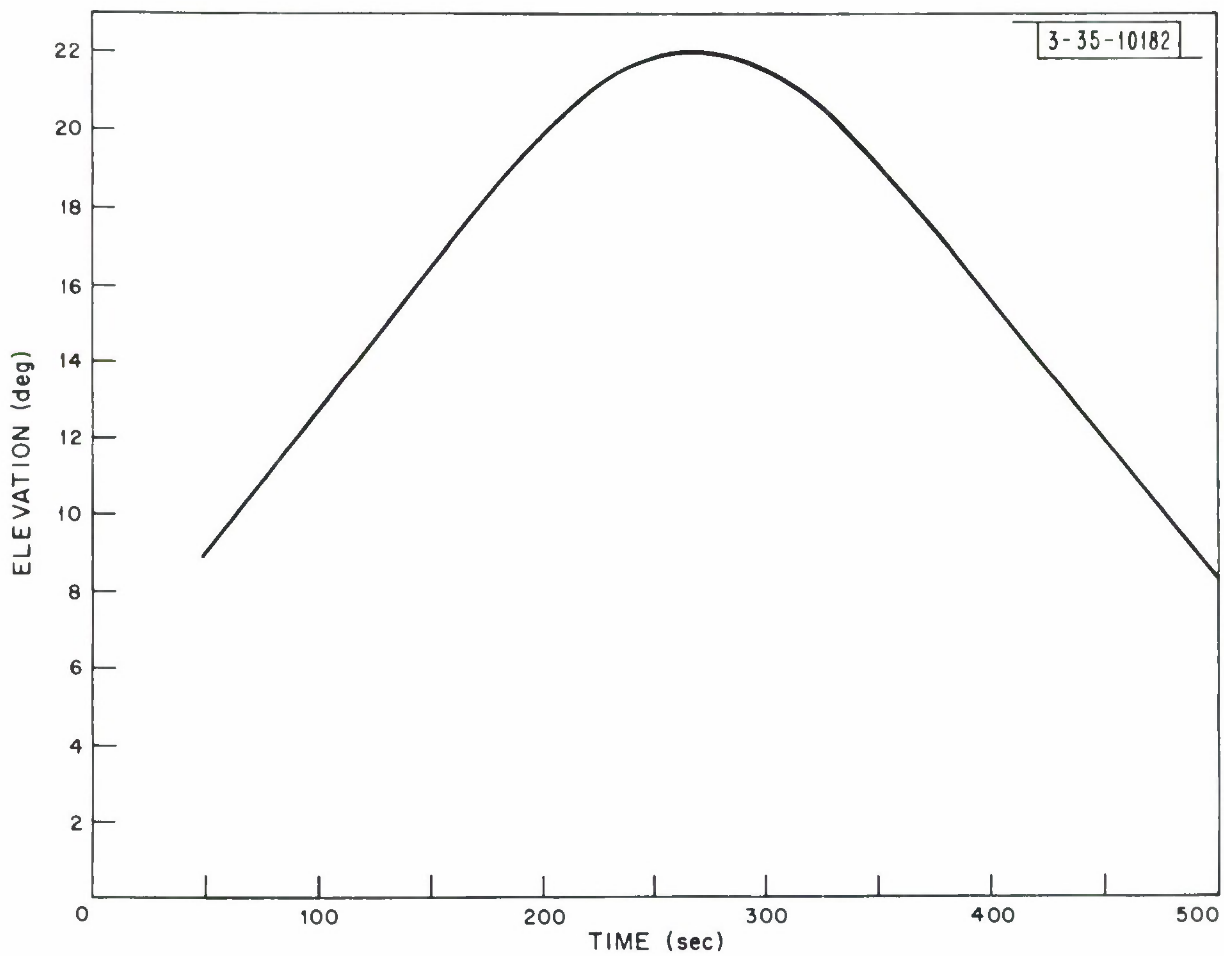

Figure 2: Elevation angle versus time for test 33.

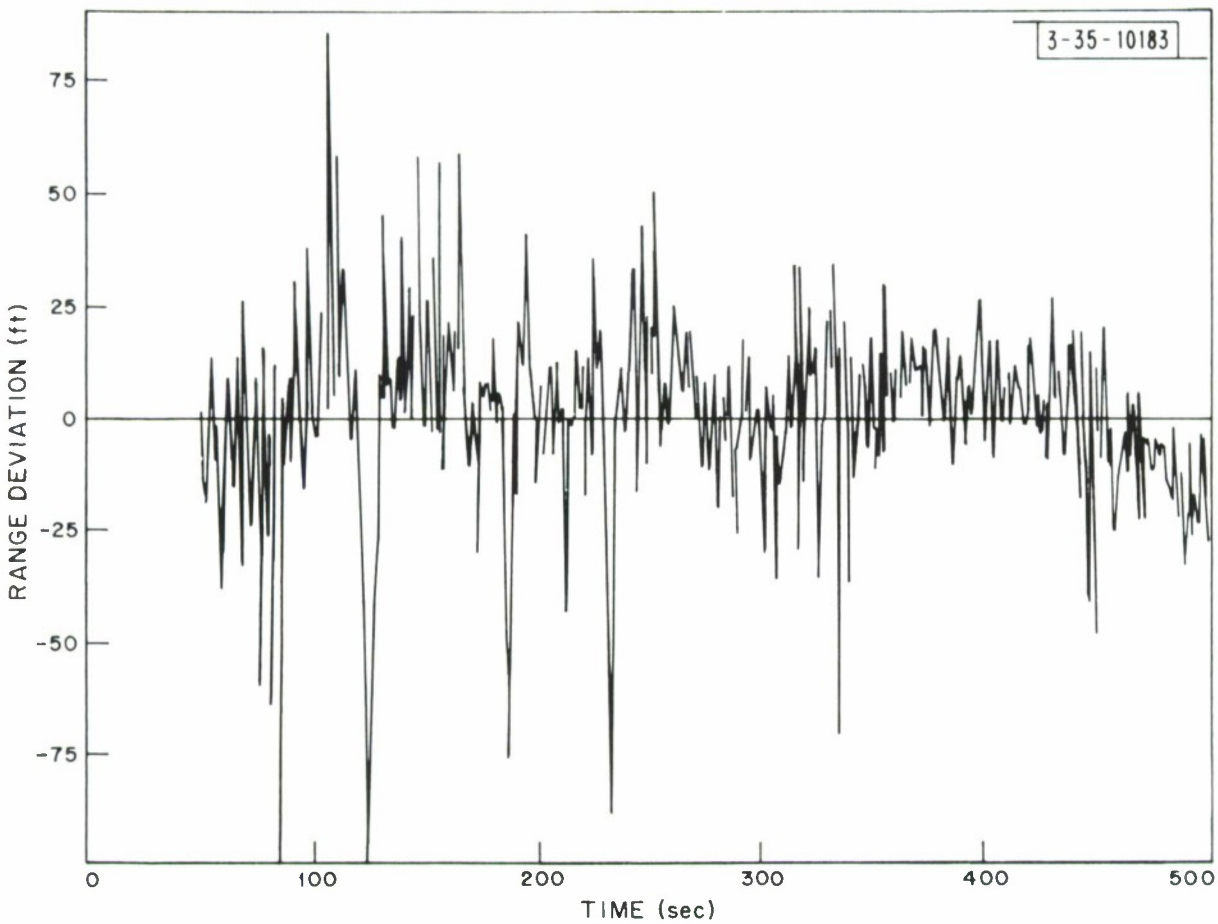

Figure 3: Range residuals versus time for test 33.

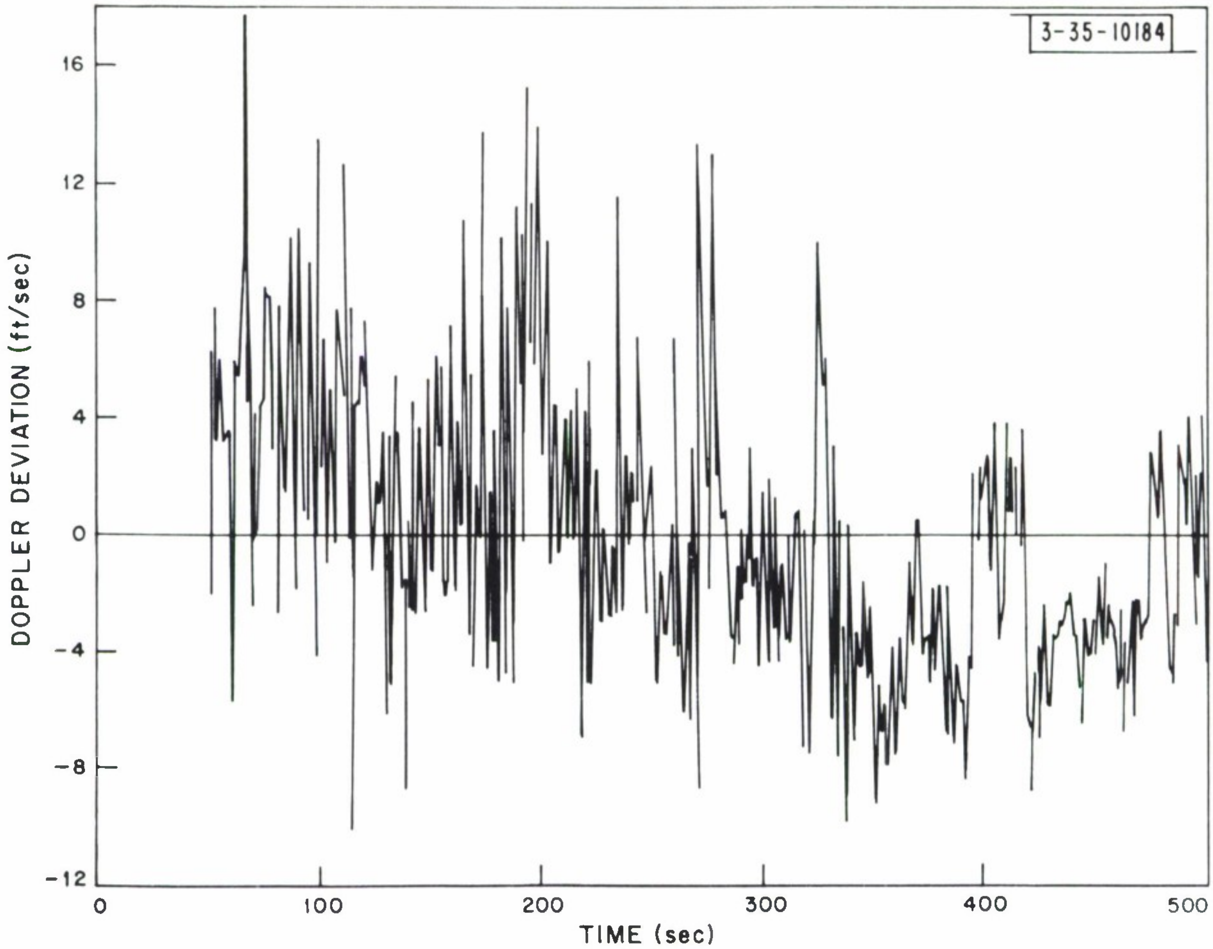

Figure 4: Doppler residuals versus time for test 33.

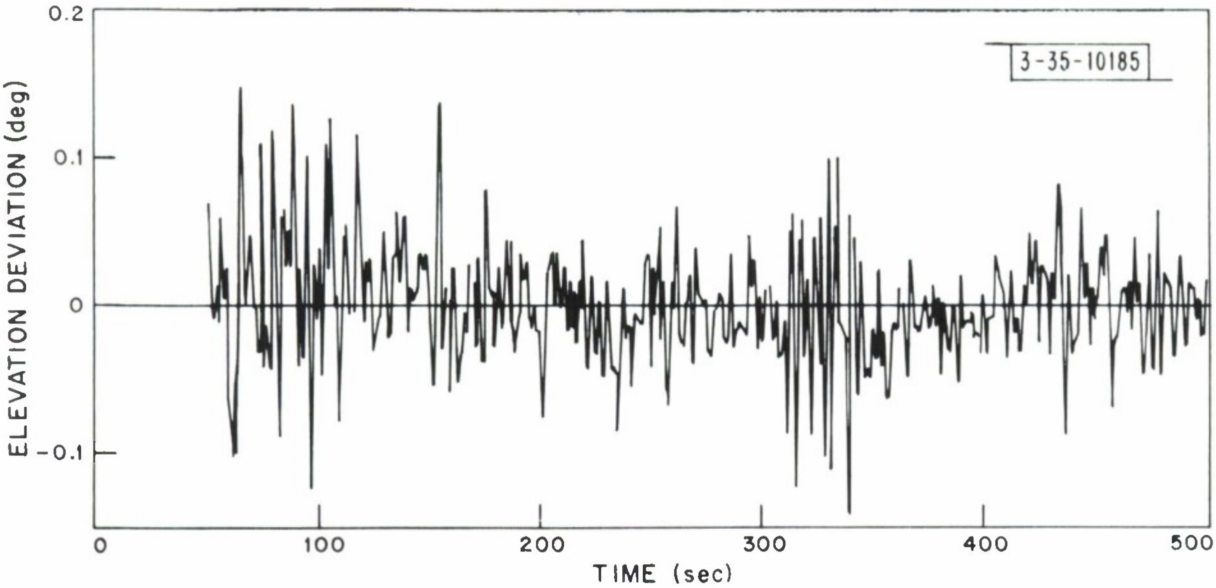

Figure 5: Elevation angle residuals versus time for test 33.

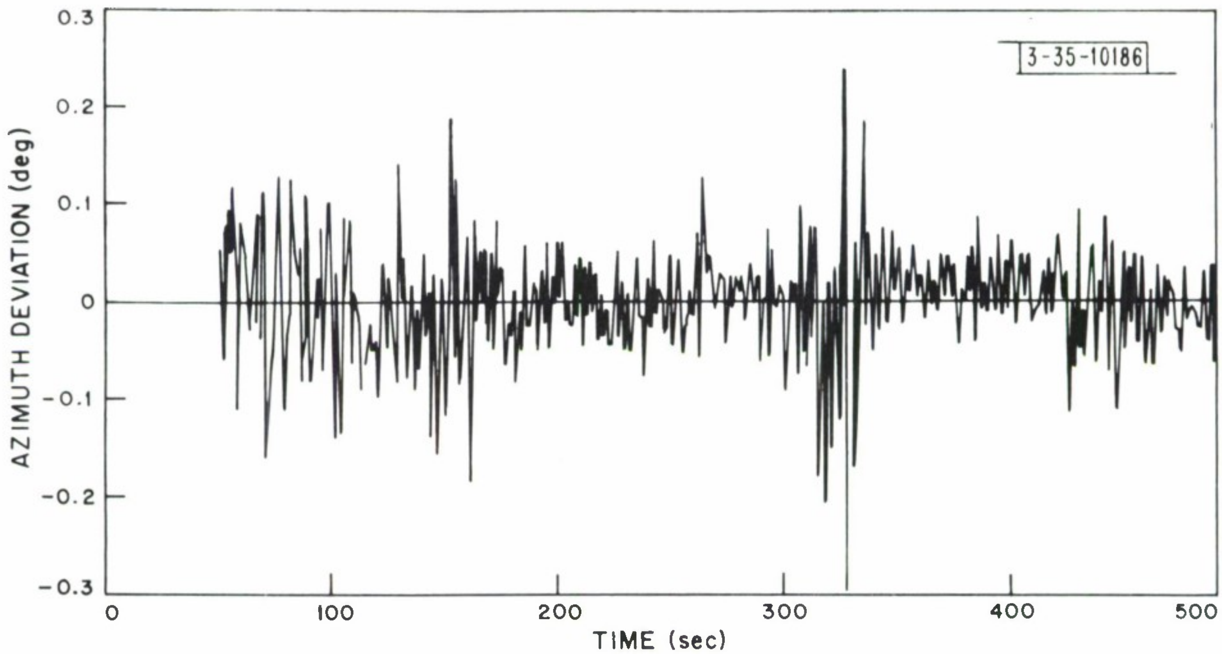

Figure 6: Azimuth angle residuals versus time for test 33.

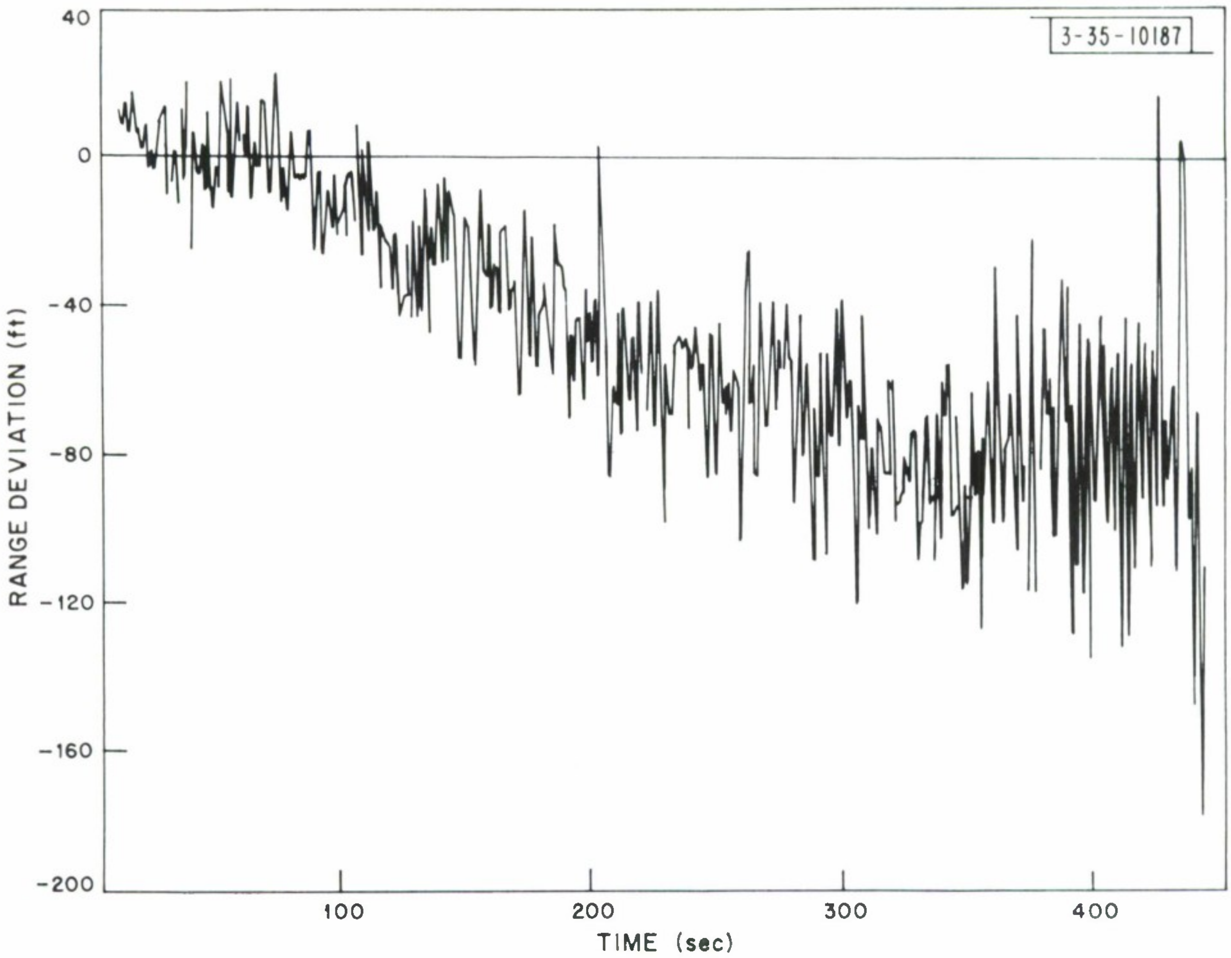

Figure 7: Range residuals versus time before convergence for test 600A.

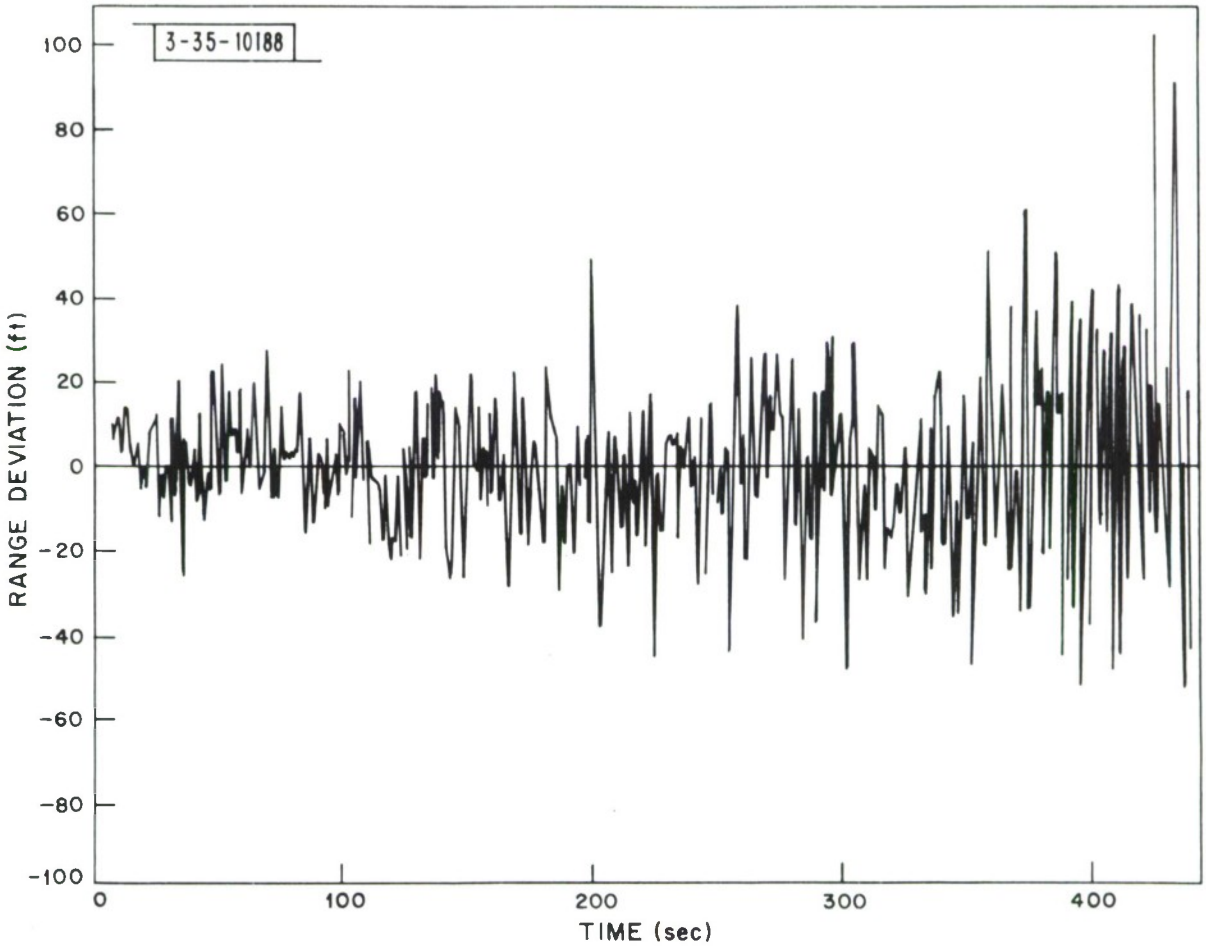

Figure 8: Range residuals versus time at convergence for test 600A.

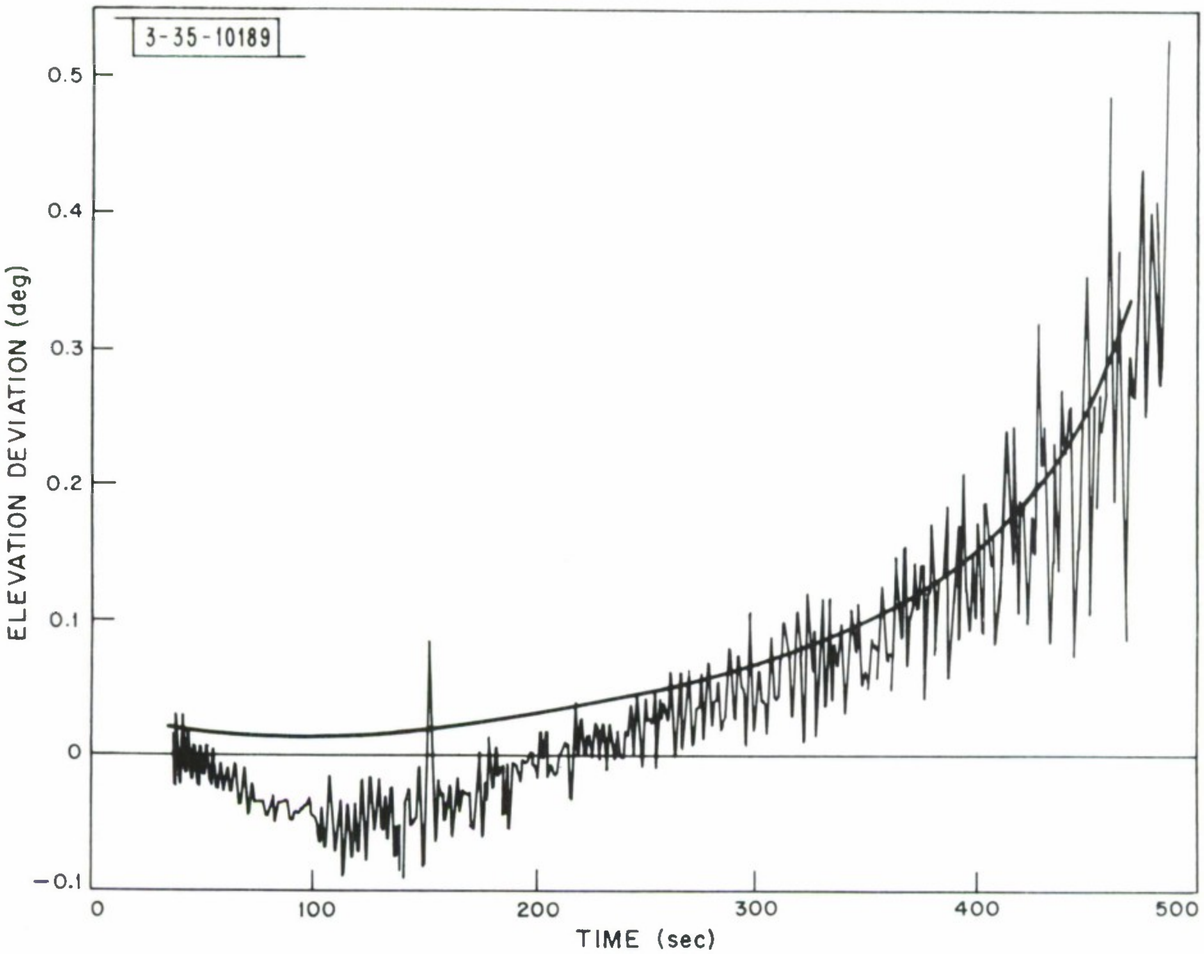

Figure 9: Elevation angle residuals as compared with tabulated refraction corrections for test 40.

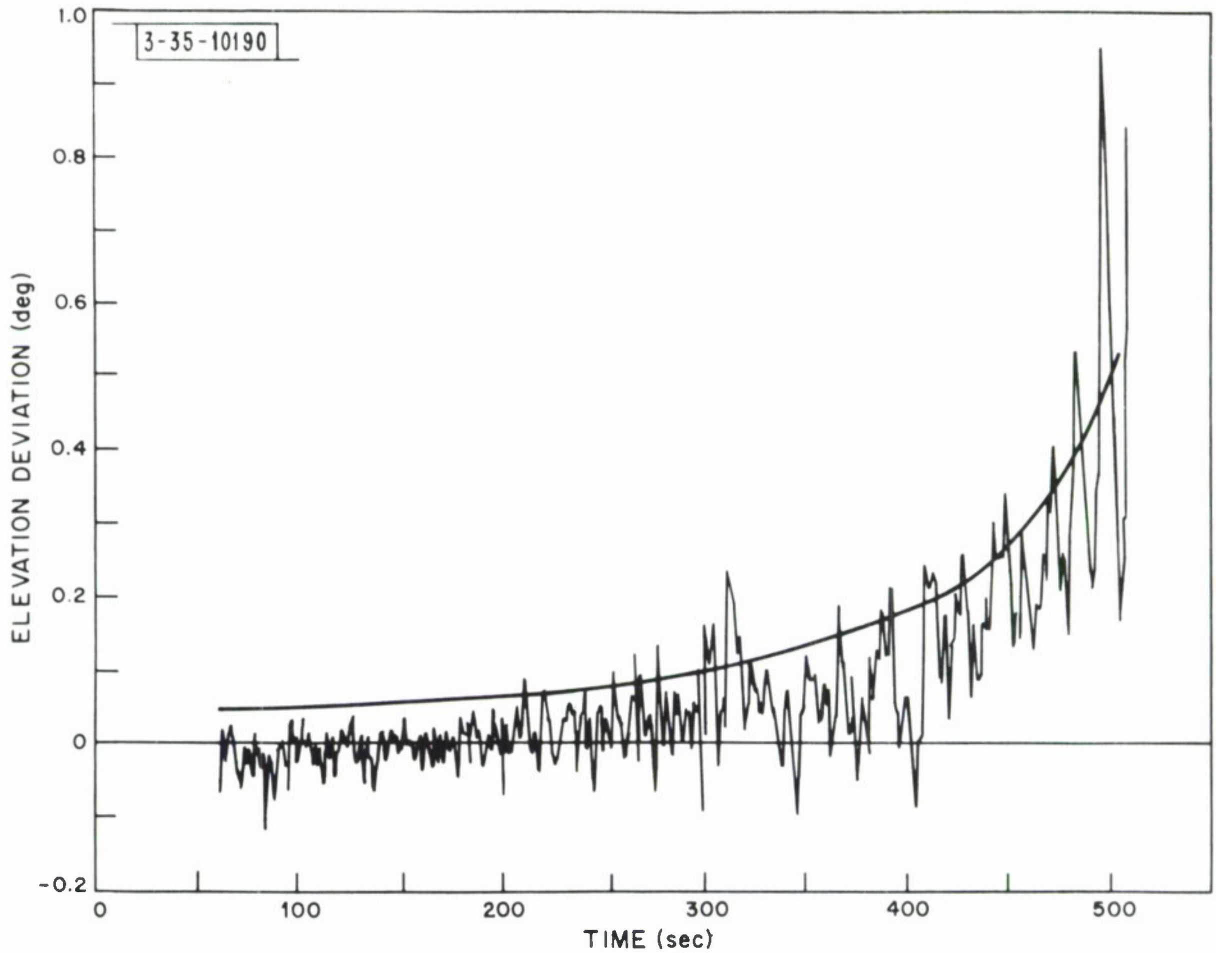

Figure 10: Elevation angle residuals as compared with tabulated refraction values for test 600B.

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13. ABSTRACT

Analysis is given for accurately determining a target's trajectory from radar data using maximum likelihood techniques. The method is incorporated in a computational program and obtains converged trajectories in 1 to 3 iterations when applied to satellite track data from the TRADEX radar. Several tracks with differing signal-to-noise ratios are presented. The average RMS residuals obtained for 300 to 450 seconds of data are 25 feet in range and 0.05 degrees in elevation and azimuth, while some tracks at high S/N resulted in angle residuals of about 0.01 degrees, equal to the pointing accuracy of TRADEX.

14. KEY WORDS

PRESS
TRADEX
satellite trajectories
signal-to-noise ratio